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Brane Inflation in the Background of D-Brane with NS B Field

Donam Youm[1]

ICTP, Strada Costiera 11, 34014 Trieste, Italy

Abstract

We study the cosmological evolution of the four-dimensional universe on the probe D3-brane in geodesic motion in the curved background of the source Dp-brane with non-zero NS B field. The Friedman equations describing the expansion of the brane universe are obtained and analyzed for various limits. We elaborate on corrections to the cosmological evolution due to nonzero NS B field.

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[1]E-mail: youmd@ictp.trieste.it

1 Introduction

Recently, theorists have actively studied the possibility that our four-dimensional universe may be a three-brane embedded in higher-dimensional spacetime, motivated by the recent proposals on solving the hierarchy problem with the large extra dimensions [1, 2, 3] and through the Randall-Sundrum (RS) scenario [4, 5, 6]. In such brane world scenario, all the matter fields live on a brane, while gravity can propagate in the bulk space. This idea is also reminiscence of the earlier proposed Horava-Witten picture [7] for the $E_8 \times E_8$ heterotic string theory, where matter is regarded as being confined on two ten-dimensional hypersurfaces on the boundaries of the eleven-dimensional spacetime.

A lot of work on the cosmological models based on the brane universe idea has been done recently. Most of the work takes an approach that the cosmological evolution of the universe is due to the time evolution of energy density on the brane, e.g. [8, 9, 10, 11, 12, 13]. (See also Ref. [14] for another approach of cosmology the RS scenario, where the canonical Wheeler-DeWitt formalism is adopted.) In this paper, we follow a different approach based on the idea that the cosmological evolution of our four-dimensional universe is due to the geodesic motion of the (probe) universe 3-brane in the curved background of other branes in the bulk [15, 16, 17, 18]. In this approach, the motion in ambient space induces cosmological expansion or contraction of the four-dimensional universe on the probe 3-brane, simulating various kinds of matter or a cosmological constant responsible for inflation [18]. Thereby, the cosmological models based on such approach are dubbed as mirage, meaning that the cosmological expansion is not due to real matter or energy density on our universe brane but due to something else. Following this approach, the cosmological evolution of the brane universe in various string theory backgrounds was studied [19, 20, 21, 22].

In this paper, we study the cosmological evolution of the brane universe on the probe D3-brane moving in the background of source D-brane with nonzero NS B field. The D-brane background with nonzero NS B field is particularly interesting because of its relevance to the recently revived noncommutative theories [23, 24, 25], thereby its possible connection to cosmology in noncommutative spacetime. We find that nonzero NS B field, or equivalently the noncommutative parameter of the brane worldvolume theories, gives rise to nontrivial corrections to the evolution of brane universe when the NS B field is electric.

The paper is organized as follows. In section 2, we review the general formalism of mirage cosmology proposed in Ref. [18]. In section 3, we study the cosmological evolution on the probe D3-brane moving in the background of the source D-brane with nonzero magnetic NS B field background. In section 4, we repeat the same analysis with the source D-brane with nonzero electric NS B field.

2 General Formalism

In the formalism of mirage cosmology proposed in Ref. [18], our four-dimensional universe is regarded as the probe D3-brane moving freely in the background of some source brane. The source brane is assumed to be not affected by the back-reaction from the probe D3-brane, as the source brane is much heavier than the probe brane. The metric of the source p-brane is parametrized as

$$ds^2_{10} = G_{\mu\nu}dx^\mu dx^\nu = g_{00}(r)dt^2 + g(r)ds^2_p + g_{rr}(r)dr^2 + g_S(r)d\Omega_{8-p}, \quad (1)$$

where $\mu,\nu = 0,1,...,9$, the line element of the unit sphere is parametrized as $d\Omega_{8-p} = h_{ij}(\varphi)d\varphi^i d\varphi^j$ $(i,j = 1,...,8-p)$ and $p \geq 3$. The action for the probe D3-brane is given by [2]

$$\begin{aligned} S &= T_3\int d^4\xi\,\mathcal{L} = T_3\int d^4\xi\,e^{-\phi}\sqrt{-\det\left(\hat{G}_{\alpha\beta}+2\pi\alpha' F_{\alpha\beta}-\hat{B}_{\alpha\beta}\right)} \\ &\quad +T_3\int d^4\xi\,\hat{A}^4 + T_3\int d^4\xi\,\hat{A}^2\wedge\hat{B}, \end{aligned} \quad (2)$$

where the hatted fields are pullbacks of the target space fields, defined for example for the case of the metric as

$$\hat{G}_{\alpha\beta} = G_{\mu\nu}\frac{\partial x^\mu}{\partial\xi^\alpha}\frac{\partial x^\nu}{\partial\xi^\beta}, \quad (3)$$

where $\alpha,\beta = 0,1,...,p$. We assume the static gauge $x^\alpha = \xi^\alpha$ for the probe action and that the transverse target space coordinates depend on the time coordinate $\xi^0 = t$, only. Note, the fields in the probe action (2) correspond to the fields generated by the source brane, as felt by the probe D3-brane. Substituting the solution for the source brane into the probe action, generally one has the following form of the probe Lagrangian density

$$\mathcal{L} = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r), \quad (4)$$

where the overdot stands for derivative with respect to t and the expressions for A, B, C and D depend on the type of the source brane.

To obtain the equations describing the geodesic motion of the probe brane, we consider the following canonical momenta and the Hamiltonian of the probe brane:

$$\begin{aligned} p_r &= \frac{\partial\mathcal{L}}{\partial\dot{r}} = -\frac{B\dot{r}}{\sqrt{A - B\dot{r}^2 - Dh_{ij}\dot{\varphi}^i\dot{\varphi}^j}}, \\ p_i &= \frac{\partial\mathcal{L}}{\partial\dot{\varphi}^i} = -\frac{Dh_{ij}\dot{\varphi}^j}{\sqrt{A - B\dot{r}^2 - Dh_{ij}\dot{\varphi}^i\dot{\varphi}^j}}, \end{aligned}$$

[2] I would like to thank R. Cai for pointing out the last term in the probe D3-brane action that was missing in the first version of the paper.

$$H = -E = p_r\dot{r} + p_i\dot{\varphi}^i - \mathcal{L} = C - \frac{A}{\sqrt{A - B\dot{r}^2 - Dh_{ij}\dot{\varphi}^i\dot{\varphi}^j}}. \tag{5}$$

Making use of the fact that the energy E and the total angular momentum $h^{ij}\varphi^i\varphi^j = \ell^2$ are conserved, from Eq. (5) one obtains the following equation describing the radial motion of the probe:

$$\dot{r}^2 = \frac{A}{B}\left(1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D}\right), \tag{6}$$

along with

$$h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2\ell^2}{D^2(C+E)^2}. \tag{7}$$

From Eq. (6), one obtains the following constraint:

$$1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D} \geq 0, \tag{8}$$

which restricts the allowed value of the scale factor a defined below.

The metric of the four-dimensional universe in the mirage cosmology scenario is given by the following induced metric on the probe D3-brane:

$$d\hat{s}^2 = (g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g ds_3^2. \tag{9}$$

Making use of Eqs. (6) and (7), one can further simplify the (t,t)-component of the induced metric. By defining the cosmic time η through

$$d\eta^2 = -(g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2, \tag{10}$$

one can put the induced metric (9) into the following form similar to the standard metric for the expanding universe:

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))ds_3^2. \tag{11}$$

Note, the three-dimensional metric ds_3^2 for the case of a source D-brane with the non-zero NS B-field is not in general that of three-dimensional flat Euclidean space, and therefore the metric (11) does not correspond to the Robertson-Walker metric for a *flat* expanding universe unlike the previous cases in Refs. [18, 19, 20, 21, 22] but rather describes the expansion of anisotropic universe. However, when the probe D3-brane is very close and very far away from the source brane, the metric ds_3^2 approaches the flat Euclidean metric, so one can apply the result of the standard flat expanding universe.

To derive an analogue of the four-dimensional Friedman equations for the expanding four-dimensional universe on the probe D3-brane, we define the scale factor a as $a^2 = g$ with the Hubble parameter defined as $H = \dot{a}/a$, where the overdot from now on stands for derivative with respect to the cosmic time η. Since the scale factor is given by $a = H^{-1/4} = (1 + \frac{Q_p}{r^{7-p}})^{-1/4}$ for the source brane solutions under consideration in this paper, the four-dimensional universe on the probe brane expands [contracts] when the probe brane falls towards [moves away from] the source brane. The equation describing the evolution of the four-dimensional universe on the probe brane is then

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{4}\dot{r}^2 \left(\frac{g'}{g}\right)^2, \tag{12}$$

where the prime stands for derivative with respect to r. The explicit expression for $\dot{r}^2$ can be found, case by case, by making use of Eqs. (6) and (10). Since the three-dimensional metric ds_3^2 in Eq. (11) is not flat Euclidean for the case under consideration in this paper, the RHS of Eq. (12) in general is not proportional to the effective matter density $\rho_{\rm eff}$ on the probe brane. However, when the probe D3-brane is very close to the source D-brane (with the non-zero NS B field), i.e. the IR regime of the dual gauge theory on the boundary, and is very far away from the source brane, for which the metric ds_3^2 approximates to the flat three-dimensional Euclidean space metric, one can set the RHS of Eq. (12) to be $\approx \frac{8\pi}{3}\rho_{\rm eff}$. We also have

$$\frac{\ddot{a}}{a} = \left(1 + \frac{g}{g'}\frac{\partial}{\partial r}\right)\frac{1}{4}\dot{r}^2\left(\frac{g'}{g}\right)^2 = \left[1 + \frac{1}{2}a\frac{\partial}{\partial a}\right]\frac{1}{4}\dot{r}^2\left(\frac{g'}{g}\right)^2 = -\frac{4\pi}{3}(\rho_{\rm eff} + 3p_{\rm eff}), \tag{13}$$

where $p_{\rm eff}$ is the effective pressure.

3 Brane Cosmology in the Background of D-Brane with the Magnetic NS B Field

In this section, we study the mirage cosmology on the probe D3-brane moving in the bulk of the source Dp-brane with nonzero magnetic NS B field. The four-dimensional universe on the probe D3-brane has the space/space noncommutativity.

The supergravity solution for the source p-brane with the rank 2 magnetic NS B field is given by [26, 27, 28, 29, 30]

$$\begin{aligned}
ds_{10}^2 &= H_p^{-\frac{1}{2}}\left[-fdt^2 + h(dx_1^2 + dx_2^2) + dx_3^2 + \cdots + dx_p^2\right] + H_p^{\frac{1}{2}}\left(f^{-1}dr^2 + r^2 d\Omega_{8-p}^2\right), \\
e^{2\phi} &= H_p^{\frac{3-p}{2}} h, \qquad B_{12} = H_p^{-1} h \tan\theta, \\
A^p_{01\dots p} &= (H_p^{-1} - 1) h\cos\theta, \qquad A^{p-2}_{03\dots p} = (H_p^{-1} - 1)\sin\theta, \\
H_p &= 1 + \frac{L_p^{7-p}}{r^{7-p}}, \qquad f = 1 - \left(\frac{r_0}{r}\right)^{7-p}, \qquad h^{-1} = \cos^2\theta + H_p^{-1}\sin^2\theta,
\end{aligned} \tag{14}$$

where $L_p^{7-p} = r_0^{7-p}\sinh^2\alpha_p$.

Substituting the solution (14) into the probe action (2), one obtains the Lagrangian density of the form (4) with the coefficients given by

$$\begin{aligned} A(r) &= g(g^2h^2+b^2)|g_{00}|e^{-2\phi}, \\ B(r) &= g(g^2h^2+b^2)g_{rr}e^{-2\phi}, \\ D(r) &= g(g^2h^2+b^2)g_S e^{-2\phi}, \end{aligned} \tag{15}$$

where

$$g_{00} = -fH_p^{-\frac{1}{2}}, \quad g = H_p^{-\frac{1}{2}}, \quad g_{rr} = f^{-1}H_p^{\frac{1}{2}}, \quad g_S = r^2 H_p^{\frac{1}{2}}, \quad b = H_p^{-1}h\tan\theta. \tag{16}$$

In the $p = 3$ case, we have the following additional term coming from the last two terms of the probe action (2):

$$C(r) = \frac{1}{\cos\theta}(1 - H_p^{-1}). \tag{17}$$

So, the equations (6) and (7) for the probe geodesic motion take the following forms:

$$\begin{aligned} \left(\frac{dr}{dt}\right)^2 &= \frac{|g_{00}|}{g_{rr}}\left[1 - \frac{|g_{00}|}{g_S}\frac{g(g^2h^2+b^2)g_S e^{-2\phi} + \ell^2}{(C+E)^2}\right], \\ h_{ij}\frac{d\varphi^i}{dt}\frac{d\varphi^j}{dt} &= \frac{g_{00}^2}{g_S^2}\frac{\ell^2}{(C+E)^2}. \end{aligned} \tag{18}$$

Making use of the equations of the motion (18), one can simplify the induced metric (9) on the probe D3-brane as follows:

$$d\hat{s}^2 = -\frac{g_{00}^2 g(g^2h^2+b^2)e^{-2\phi}}{(C+E)^2}dt^2 + g ds_3^2, \tag{19}$$

where $ds_3^2 = hdx_1^2 + hdx_2^2 + dx_3^2$. Then, the cosmic time η, with which induced metric (19) takes the standard form (11) of an expanding universe, is defined as

$$d\eta = \frac{|g_{00}|g^{\frac{1}{2}}(g^2h^2+b^2)^{\frac{1}{2}}e^{-\phi}}{|C+E|}dt. \tag{20}$$

Therefore, the four-dimensional Friedman equation (12) takes the following form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{(C+E)^2 g_S e^{2\phi} - |g_{00}|[g_S g(g^2h^2+b^2) + \ell^2 e^{2\phi}]}{4|g_{00}|g_{rr}g_S g(g^2h^2+b^2)}\left(\frac{g'}{g}\right)^2. \tag{21}$$

We now express the RHS of the Friedman equation (21) in terms of the scale factor a. This is achieved by substituting Eqs. (16) and (17) into Eq. (21) and making use of the relation $r^{7-p} = L_p^{7-p} a^4/(1-a^4)$. For the probe D3-brane at an arbitrary distance from the source brane, the Friedman equation (21) takes the form:

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= \tfrac{(p-7)^2}{16} L_p^{-2} a^6 \left(\tfrac{1-a^4}{a^4}\right)^{\frac{2(8-p)}{7-p}} \Big[(E\cos\theta)^2 a^{2(p-5)} \\ &\quad - \tfrac{(L_p^{7-p}+r_0^{7-p})a^4 - r_0^{7-p}}{L_p^{7-p}} \left\{1 + \tfrac{(\ell\cos\theta)^2}{L_p^2} \left(\tfrac{1-a^4}{a^4}\right)^{\frac{2}{7-p}} a^{2(p-5)}\right\}\Big], \end{aligned} \tag{22}$$

for $p \neq 3$, and

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= L_3^{-2} a^6 \left(\tfrac{1-a^4}{a^4}\right)^{\frac{5}{2}} \Big[\left(E\cos\theta + 1 - a^4\right)^2 a^{-4} \\ &\quad - \tfrac{(L_3^4+r_0^4)a^4 - r_0^4}{L_3^4} \left\{1 + \tfrac{(\ell\cos\theta)^2}{L_3^2} \left(\tfrac{1-a^4}{a^4}\right)^{\frac{1}{2}} a^{-4}\right\}\Big], \end{aligned} \tag{23}$$

for $p = 3$. The constraint (8) that the RHS of the Friedman equations (22) and (23) are non-negative limits the size of the scale factor to be $0 \leq a \leq 1$. In particular, when the probe D3-brane is in the near horizon region of the source brane, for which $H_p \approx L_p^{7-p}/r^{7-p}$, the radial coordinate is related to the scale factor as $r^{7-p} \approx L_p^4 a^4$. So, the Friedman equation (21) becomes

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &\approx \tfrac{(p-7)^2}{16} L_p^2 a^{-2\frac{p-11}{p-7}} \Big[(E\cos\theta)^2 a^{2(p-5)} \\ &\quad - \tfrac{L_p^{7-p}a^4 - r_0^{7-p}}{L_p^{7-p}} \left\{1 + \tfrac{(\ell\cos\theta)^2}{L_p^2} a^{\frac{8}{p-7}+2(p-5)}\right\}\Big], \end{aligned} \tag{24}$$

for $p \neq 3$, and

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &\approx L_3^2 a^{-4} \Big[\left(E\cos\theta + 1 - a^4\right)^2 a^{-4} \\ &\quad - \tfrac{L_3^4 a^4 - r_0^4}{L_3^4} \left(1 + \tfrac{(\ell\cos\theta)^2}{L_3^2} a^{-6}\right)\Big], \end{aligned} \tag{25}$$

for $p = 3$. If we consider just the near-horizon geometry (for an arbitrary r) of the source brane, the constraint (8) does not restrict the allowed value of the scale factor a, which thereby taking $0 \leq a < \infty$.

We see from the above Friedman equations that the cosmological evolution on the probe D3-brane does not have any qualitative difference from the case without NS B-field [3] studied in [18]. Namely, the nonzero magnetic NS B field has an effect of just weakening the energy E and the total angular momentum ℓ in the Friedman equations by the factor $\cos\theta$, thereby slowing down inflation of the brane universe. So, just like the case without the NS B field, when the probe D3-brane is very close to the source brane (i.e. $a \ll 1$), the Friedman equation is dominated by the term

[3] It is also observed in Refs. [31, 32, 33] that there is no qualitative difference in thermodynamics of a probe D-brane between the cases with and without the magnetic NS B-field.

$\sim a^{-2(p-11)/(p-7)+2(p-5)}$ for $\ell = 0$ and by the term $\sim a^{-2(p-11)/(p-7)+8/(p-7)+2(p-5)}$ for $\ell \neq 0$ when $p \leq 5$, and by the term $\sim a^{-10}$ for $\ell = 0$ and by the term $\sim a^{-16}$ for $\ell \neq 0$ when $p = 6$. When the NS B field is very large, i.e. $\theta \approx \pi/2$, the Friedman equation is dominated by term $\sim a^{-2(p-11)/(p-7)}$, as the rest terms are suppressed by the factor $\cos^2\theta \approx 0$. Note, when the perfect fluids, which the cosmological models assume matter and energy in the universe to be, satisfy the equation of state of the form

$$p = w\rho, \tag{26}$$

where w is time-independent constant, the conservation of energy equation can be integrated to give

$$\rho \propto a^{-3(1+w)}. \tag{27}$$

In the region very close to the source brane, the induced metric (11) approximates to the Robertson-Walker metric for a flat expanding universe, for which case the RHS of the Friedman equation is approximately $\frac{8\pi}{3}\rho_{\rm eff}$. So, the $p = 3$ case, for which $\rho_{\rm eff} \sim a^{-4}$ (or $w = 1/3$) for the large NS B field or for the finite B field with $\ell = 0$, simulates radiation dominated universe. And the $p = 3$ case with finite NS B field and $\ell \neq 0$ corresponds to $\rho_{\rm eff} = p_{\rm eff}$, which is characteristic of a massless scalar. Note, the causality restricts w in the equation of state (26) to be $|w| \leq 1$. So, the $p = 6$ case, for which $\rho_{\rm eff} \sim a^{-10}$ (or $w = 7/3$) with $\ell = 0$ and $\rho_{\rm eff} \sim a^{-16}$ (or $w = 13/3$) with $\ell \neq 0$, couldn't possibly have been attained by real matter on the probe brane. For the $a \approx 1$ case, i.e. when the probe D3-brane is far away from the source brane, the leading order behavior of the Friedman equation in terms of $\epsilon = 1 - a^4 \approx 0$ is given by

$$\dot\epsilon \approx \frac{|p-7|}{16L_p}\sqrt{E^2\cos^2\theta - 1}\,\epsilon^{\frac{8-p}{7-p}} \quad\Longrightarrow\quad \epsilon \approx (4L_p)^{7-p}(E^2\cos^2\theta - 1)^{\frac{p-7}{2}}\eta^{p-7}, \tag{28}$$

similarly as in the case without the NS B field. So, the $p < 7$ case corresponds to the asymptotically flat solution. The subleading corrections (with additional positive powers of ϵ) to Eq. (28) are suppressed by the factor $\cos^2\theta$. From Eq. (28), one can also see that the maximum allowed value of θ is given by $\cos\theta = 1/E$ at later stage of inflation (for which $a \approx 1$).

4 Brane Cosmology in the Background of D-brane with the Electric NS B Field

In this section, we study the mirage cosmology on the probe D3-brane moving in the bulk of the source Dp-brane with nonzero electric NS B field. The four-dimensional universe on the probe D3-brane has the space/time noncommutativity.

The supergravity solution for the source p-brane with the rank 2 electric NS B field is given by [34, 29]

$$
\begin{aligned}
ds^2_{10} &= H_p^{-\frac{1}{2}}\left[h'(-f dt^2 + dx_1^2) + dx_2^2 + \cdots + dx_p^2\right] + H_p^{\frac{1}{2}}\left(f^{-1}dr^2 + r^2 d\Omega^2_{8-p}\right), \\
e^{2\phi} &= H_p^{\frac{3-p}{2}} h', \qquad B_{t1} = H_p^{-1} h' \tanh\theta', \\
A^p_{01\ldots p} &= (1-H_p^{-1})h'\cosh\theta', \qquad A^{p-2}_{2\ldots p} = (1-H_p^{-1})\sinh\theta',
\end{aligned} \tag{29}
$$

where $Q_p = r_0^{7-p}\sinh^2\alpha_p$ and $h'^{-1} = \cosh^2\theta' - H_p^{-1}\sinh^2\theta'$.

Substituting the solution (29) into the probe action (2), one obtains the Lagrangian density of the form (4) with the coefficients given by

$$
\begin{aligned}
A(r) &= (g^3 h'|g_{00}| - b^2 g^2)e^{-2\phi}, \\
B(r) &= g^3 h' g_{rr} e^{-2\phi}, \\
D(r) &= g^3 h' g_S e^{-2\phi},
\end{aligned} \tag{30}
$$

where

$$
\begin{aligned}
g_{00} &= -f h' H_p^{-\frac{1}{2}}, \qquad g = H_p^{-\frac{1}{2}}, \qquad g_{rr} = f^{-1}H_p^{\frac{1}{2}}, \\
g_S &= r^2 H_p^{\frac{1}{2}}, \qquad b = H_p^{-1} h' \tanh\theta'.
\end{aligned} \tag{31}
$$

In the $p = 3$ case, we have the following additional term coming from the last two terms in the probe action (2):

$$
C(r) = \frac{1}{\cosh\theta'}(1 - H_p^{-1}). \tag{32}
$$

So, the equations describing the probe geodesic motion take the following forms:

$$
\begin{aligned}
\left(\frac{dr}{dt}\right)^2 &= \frac{gh'|g_{00}| - b^2}{gh' g_{rr}}\left[1 - \frac{(gh'|g_{00}| - b^2)(g^3 h' g_S e^{-2\phi} + \ell^2)}{gh' g_S (C+E)^2}\right], \\
h_{ij}\frac{d\varphi^i}{dt}\frac{d\varphi^j}{dt} &= \frac{(gh'|g_{00}| - b^2)^2}{g^2 h'^2 g_S^2}\frac{\ell^2}{(C+E)^2}.
\end{aligned} \tag{33}
$$

Making use of the equations of motion (33), one can bring the induced metric (9) on the probe D3-brane to the following form:

$$
d\hat{s}^2 = -\left[\frac{(gh'|g_{00}| - b^2)^2 g e^{-2\phi}}{h'(C+E)^2} + b^2 g^{-1} h'^{-1}\right] dt^2 + g ds_3^2, \tag{34}
$$

where $ds_3^2 = h' dx_1^2 + dx_2^2 + dx_3^2$. So, the cosmic time η is defined through

$$
d\eta = \sqrt{\frac{(gh'|g_{00}| - b^2)^2 g e^{-2\phi}}{h'(C+E)^2} + b^2 g^{-1} h'^{-1}}\, dt. \tag{35}
$$

Therefore, the four-dimensional Friedman equation (12) takes the following form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{(h'|g_{00}|g - b^2)\left[h'g_S g(C+E)^2 - (h'|g_{00}|g - b^2)(h'g_S g^3 e^{-2\phi} + \ell^2)\right]}{4h'g_{rr}g_S g\left[(h'|g_{00}|g - b^2)^2 g^2 e^{-2\phi} + b^2(C+E)^2\right]}\left(\frac{g'}{g}\right)^2. \tag{36}$$

We now express the RHS of the Friedman equation (36) in terms of the scale factor a. For this purpose, we substitute Eqs. (31) and (32) into Eq. (36). For the probe D3-brane at an arbitrary distance from the source brane, for which the radial coordinate is related to the scale factor as $r^{7-p} = Q_p a^4/(1-a^4)$, the Friedman equation (36) takes the form

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= \frac{(7-p)^2 Q_p a^6[(Q_p+r_0^{7-p})a^4 - r_0^{7-p}]}{16[Q_p a^4/(1-a^4)]^{\frac{2(8-p)}{7-p}}}\left[\frac{(Q_p+r_0^{7-p})a^4 - r_0^{7-p}}{Q_p} - a^8\tanh^2\theta'\right] \\ &\quad\times\frac{\frac{E^2}{\cosh^2\theta' - a^4\sinh^2\theta'} - \left\{\frac{(Q_p+r_0^{7-p})a^4 - r_0^{7-p}}{Q_p} - a^8\tanh^2\theta'\right\}\left\{a^{2(5-p)} + \ell^2(\frac{1-a^4}{Q_p a^4})^{\frac{2}{7-p}}\right\}}{\left\{\frac{(Q_p+r_0^{7-p})a^4 - r_0^{7-p}}{Q_p} - a^8\tanh^2\theta'\right\}^2 a^{2(5-p)} + E^2\frac{a^8\tanh^2\theta'}{\cosh^2\theta' - a^4\sinh^2\theta'}},\end{aligned} \tag{37}$$

for $p \neq 3$, and

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= \frac{Q_3 a^6[(Q_3+r_0^4)a^4 - r_0^4]}{[Q_3 a^4/(1-a^4)]^{\frac{5}{2}}}\left[\frac{(Q_3+r_0^4)a^4 - r_0^4}{Q_3} - a^8\tanh^2\theta'\right] \\ &\quad\times\frac{\left(E + \frac{1-a^4}{\cosh\theta'}\right)^2\frac{1}{\cosh^2\theta' - a^4\sinh^2\theta'} - \left\{\frac{(Q_3+r_0^4)a^4 - r_0^4}{Q_3} - a^8\tanh^2\theta'\right\}\left\{a^4 + \ell^2(\frac{1-a^4}{Q_3 a^4})^{\frac{1}{2}}\right\}}{\left\{\frac{(Q_3+r_0^4)a^4 - r_0^4}{Q_3} - a^8\tanh^2\theta'\right\}^2 a^4 + \left(E + \frac{1-a^4}{\cosh\theta'}\right)^2\frac{a^8\tanh^2\theta'}{\cosh^2\theta' - a^4\sinh^2\theta'}},\end{aligned} \tag{38}$$

for $p = 3$. As in the case of the source brane with the magnetic NS B field, the constraint (8) that the RHS of the Friedman equations (37) and (38) are non-negative limits the size of the scale factor to be $0 \leq a \leq 1$. When the probe D3-brane is in the near-horizon region of the source brane, in which case $r^{7-p} \approx Q_p a^4$, the Friedman equation becomes

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &\approx \frac{(7-p)^2 a^2(Q_p a^4 - r_0^{7-p})}{16(Q_p a^4)^{\frac{9-p}{7-p}}}\left[\frac{Q_p a^4 - r_0^{7-p}}{Q_p} - a^8\tanh^2\theta'\right] \\ &\quad\times\frac{\frac{E^2}{\cosh^2\theta' - a^4\sinh^2\theta'} - \left\{\frac{Q_p a^4 - r_0^{7-p}}{Q_p} - a^8\tanh^2\theta'\right\}\left\{a^{2(5-p)} + \ell^2(Q_p a^4)^{\frac{2}{p-7}}\right\}}{\left\{\frac{Q_p a^4 - r_0^{7-p}}{Q_p} - a^8\tanh^2\theta'\right\}^2 a^{2(5-p)} + E^2\frac{a^8\tanh^2\theta'}{\cosh^2\theta' - a^4\sinh^2\theta'}},\end{aligned} \tag{39}$$

for $p \neq 3$, and

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &\approx \frac{a^2(Q_3 a^4 - r_0^4)}{(Q_3 a^4)^{\frac{3}{2}}}\left[\frac{Q_3 a^4 - r_0^4}{Q_3} - a^8\tanh^2\theta'\right] \\ &\quad\times\frac{\left(E + \frac{1-a^4}{\cosh\theta'}\right)^2\frac{1}{\cosh^2\theta' - a^4\sinh^2\theta'} - \left\{\frac{Q_3 a^4 - r_0^4}{Q_3} - a^8\tanh^2\theta'\right\}\left\{a^4 + \ell^2(Q_3 a^4)^{-\frac{1}{2}}\right\}}{\left\{\frac{Q_3 a^4 - r_0^4}{Q_3} - a^8\tanh^2\theta'\right\}^2 a^4 + \left(E + \frac{1-a^4}{\cosh\theta'}\right)^2\frac{a^8\tanh^2\theta'}{\cosh^2\theta' - a^4\sinh^2\theta'}},\end{aligned} \tag{40}$$

for $p = 3$.

Unlike the case of the source D-brane with the magnetic NS B field, the electric NS B field modifies the Friedman equations nontrivially. When the probe D3-brane is very close to the source brane, the θ'-dependent terms give rise to infinite series of subleading terms to $(\dot{a}/a)^2$. [The leading order behavior of the Friedman equations for the $a \ll 1$ case is the same as the magnetic NS B field case.] These subleading terms become important as a increases, and finally the cosmological evolution becomes qualitatively different from the case without the NS B field when a is no longer close to zero. When the probe D-brane is far away from the source brane (i.e. the $a \approx 1$ case), the leading order behavior of the Friedman equation is still given by

$$\dot{\epsilon} \approx \frac{|p-7|}{16} Q_p^{\frac{1}{p-7}} \sqrt{E^2 - 1}\,\epsilon^{\frac{8-p}{7-p}} \quad \Longrightarrow \quad \epsilon \approx 4^{7-p} Q_p (E^2-1)^{\frac{p-7}{2}} \eta^{p-7}, \tag{41}$$

similarly as in the magnetic NS B field case, but the nonzero electric NS B field produces infinite series of subleading corrections [4] to Eq. (41), unlike the magnetic NS B field case, for which the nonzero NS B field just suppresses the already existing subleading terms by the factor of $\cos^2\theta$. Unlike the case of magnetic NS B field, such infinite series of subleading corrections to Eq. (41) do not get suppressed as the electric B field increases, i.e. as θ' increases.

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